COALESCENCE OF GAS BUBBLES IN A SOUND FIELD

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Acoustic gas cavitation is characterized by the appearance of sheets of visually observable bubbles in a fluid [1]. Large bubbles of visible dimensions ($R_0 \sim 10^{-4}$-10^{-3} m) grow from microbubbles, cavitation seeds, under the effect of a "rectified" diffusion mechanism [2, 3]. As the bubble grows, the concentration of the gas dissolved in the fluid is reduced in an increasingly extended domain. The regime setting in during overlapping of the domains "laid waste" by the adjacent bubbles, does not correspond to the model of independent nuclei [2, 3] and requires further study. The influence of the varying gas content on the dynamics of growth (and dissolution) of cavitation bubbles is discussed in this paper. Conditions are mentioned for the occurrence of an asymptotically universal bubble size distribution.

The change in concentration of a gas dissolved in a fluid is described by the diffusion equation

$$\frac{\partial c}{\partial t} + (\mathbf{v}\cdot\nabla)c = D\Delta c, \tag{1}$$

where D is the diffusion coefficient and $\mathbf{v}$ is the velocity of fluid motion.

Let us examine the characteristic time scales existing in the problem: the period of the sound field T_1, the smallest scale by assumption, $T_2 = n^{-2/3}D^{-1}$ is the mean time of gas molecule diffusion motion between bubbles (n is the bubble concentration), T_3 is the characteristic time of the change in gas content (an explicit expression for this scale will be given below (6) in the solution of the problem). The following scale hierarchy is typical: $T_1 \ll T_2 \ll T_3$. The physical meaning of the condition $T_1 \ll T_2$ is smallness of the diffusion wavelength $\sqrt{DT_1}$ as compared with the mean spacing between bubbles $n^{-1/3}$. The inequality $T_2 \ll T_3$ implies that the deviation of the gas content from the mean value is small and, therefore, the diffusion fluxes in the space between bubbles are small. The smallness of the bubble size as compared with the distance between them ($R_0 \ll n^{-1/3}$) is also assumed.

After taking the average over the period of the sound field (1) reduces to

$$\langle(\mathbf{v}\cdot\nabla)c\rangle = D\Delta\langle c\rangle,$$

within the limits of the oscillating diffusion layer near the bubble, where $\langle\ldots\rangle$ denotes taking the average over the period of the field. We limit ourselves to the consideration of fluids of large viscosity, which permits microfluxes and translational bubble motion not to be taken into account. A quasistationary concentration distribution

$$D\Delta\langle c\rangle = 0, \quad \langle c\rangle = \tilde{c} + \sum_i \frac{\tilde{c}_i}{|\mathbf{r}-\mathbf{r}_i|}$$

is established in the times $t \gg T_2$ in the space between the bubbles (outside the oscillating diffusion layers), where $\mathbf{r}_i$ is the coordinate of the center of the i-th bubble. The long-range nature $1/|\mathbf{r}-\mathbf{r}_i|$ of the influence of the individual bubbles on the dissolved gas concentration permits the introduction of a mean (self-consistent) field on concentration $\langle c\rangle = \bar{c}(t/T_3)$.

In the neighborhood ($\sqrt{DT_1} < |\mathbf{r}-\mathbf{r}_i| < n^{-1/3}$) of the i-th bubble $\langle c\rangle = \bar{c}(t/T_3) + \tilde{c}_i/|\mathbf{r}-\mathbf{r}_i|$ The equation describing the growth of a single bubble here conserves its form [3] and only the value considered constant earlier for the gas content at a large distance from the bubble c_∞ is replaced by a variable mean value $\bar{c}(t/T_3)$:

$$\frac{dR_0}{dt} = \frac{Dc_0}{\rho_g}\frac{1}{R_0}\frac{\langle R/R_0\rangle}{(1+4\sigma/3p_\infty R_0)}\left(\frac{\bar{c}(t/T_3)}{c_0} - \frac{\langle (R/R_0)^{4-3\gamma}p_0\rangle}{\langle (R/R_0)^4\rangle p_\infty}\right), \tag{2}$$

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where $p_0 = p_\infty + 2\sigma/R_0$, p_∞ is the equilibrium value of the pressure far from the bubble, R_0 is the equilibrium radius of the bubble, σ is the surface tension coefficient, c_0 is the equilibrium value of the gas content (the mass density), ρ_g is the gas equilibrium density for fixed values of p_∞ and the temperature. The running value of the bubble radius R is found from the solution of the Rayleigh–Plesset equation

$$R\ddot{R} + \frac{3}{2}\dot{R}^2 + \rho_0^{-1}\{p_0[1-(R_0/R)^{3\gamma}] - p_m\cos\omega t + \rho_0 R_0\omega_0 b\dot{R}\} = 0.$$

Calculations are performed to the accuracy of second order terms (inclusive) in the sound wave amplitude, ρ_0 is the fluid density, γ is the polytropic index, $\omega_0{}^2 = 1/\rho_0 R_0{}^2(3\gamma p_0 - 2\sigma/R_0)$ is the natural frequency of the monopolar bubble vibrations, b is the damping constant that effectively takes account of the dissipative processes of viscosity, heat conduction, and radiation losses, and p_m, ω are the sound wave amplitude and frequency.

The time change of $\bar{c}$ is determined from the mass conservation law for the dissolved gas

$$\frac{d\bar{c}}{dt} + \frac{d}{dt}\rho_2\int dR_0\frac{4\pi}{3}R_0^3 g(R_0, t) = 0. \tag{3}$$

The bubble size distribution function $f(R_0, r)$ satisfies the continuity equation in the dimension space

$$\frac{\partial g}{\partial t} + \frac{\partial}{\partial R_0}(v_{R_0}g) = 0, \quad v_{R_0} = \dot{R}_0. \tag{4}$$

The awkward form of (2) makes a general analysis of the system (2)-(4) difficult; consequently, we limit ourselves below to a discussion of a particular case that allows an exact analytic solution. If the surface tension forces are small compared with the hydrostatic pressure and the sound field has not too high-frequency, so that the bubbles are preresonant in the whole stage under consideration, (2) acquires the simplified form

$$\dot{R}_0 = \frac{Dc_0}{\rho_g R_0}\left[\frac{\bar{c}(t/T_3) - c_0}{c_0} - \frac{2\sigma}{R_0 p_\infty} + \frac{2}{3\gamma}\left(\frac{p_m}{p_\infty}\right)^2\right]. \tag{5}$$

In the approximation taken, the influence of the "rectified" diffusion process is equivalent to an increase in the mean concentration of the dissolved gas. The system (3)-(5) here differs from the Lifshits–Slezov coalescence equations [4, 5] just by notation, and substantially describes the process of a new phase falling out of the supersaturate solution. The existence of a critical radius, determined from the condition $\dot{R}_0 = 0$ (see (5), $R_* = 2\sigma/p_\infty[(\bar{c}(t/T_3 - c_0)/c_0 + 2/3\gamma(p_m/p_\infty)^2])$ result in the fact that the subcritical bubbles ($R_0 < R_*$) are dissolved while the postcritical ($R_0 > R_*$) grow. Diminution of the effective supersaturation results in an increase in the critical radius, hence the smallest of the already available bubbles become subcritical and start to be dissolved. In this stage the dissolution of the fine bubbles begins to play a governing role in the growth process of the large-scale postcritical bubbles.

The self-similar solution found in [4, 5] at the times $t > T_3$ leads to the following results:

the gas content diminishes

$$\bar{c}(t/T_3) = c_0 - 2c_0/3\gamma(p_m/p_\infty)^2 + [c_*(0) - c_0 + 2c_0/3\gamma(p_m/p_\infty)^2](t/T_3)^{-1/3};$$

since the critical radius is determined by the mean effective supersaturation, a power-law growth occurs

$$R_*(t) = R_*(0)\left(\frac{4}{9}\frac{t}{T_3}\right)^{1/3};$$

the number of bubbles diminishes because the postcritical bubbles grow mainly because of the subcritical

$$n(t) = \int dR_0 g(R_0, t) \simeq 0{,}4\varkappa(T_3/t);$$

the mean bubble radius agrees with the critical value

$$\bar{R}(t) = \int dR_0 g(R_0, t) R_0 \simeq R_*(t);$$

this occurs to a considerable degree because of the narrowness of the distribution function

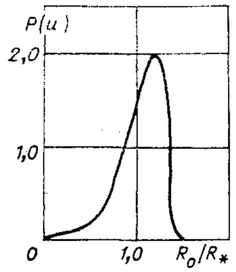

Fig. 1

$$g(R_0, t) = \frac{1}{R_*(t)}\left(\frac{R_*(0)}{R_*(t)}\right)^3 0{,}9\varkappa P(u), \quad u \equiv R_0/R_*(t),$$

$$P(u) = \frac{3^4 e}{2^{5/3}}\,\frac{u^2 \exp[-1/(1-2u/3)]}{(u+3)^{7/3}(3/2-u)^{11/3}}, \quad u < 3/2; \qquad P(u) = 0, \ u > 3/2$$

(the explicit form of the dependence P(u) is presented in Fig. 1 [5], here $\kappa = 4\pi R_*^3(0)/3Q$ is the effective initial supersaturation, and $Q = (4\pi/3)\rho_g \int dR_0 \times g(R_0, 0) R_0^3 + \bar{c}(0) - c_0 + (2c_0/3\gamma)(p_m/p_\infty)^2$.

The characteristic time of build-up of the self-similar distribution is

$$T_3 = (2\sigma/p_\infty)^2 \left\{ D \frac{c_0}{\rho_g} \left[\frac{\bar{c}(0) - c_0}{c_0} + \frac{2}{3\gamma}\left(\frac{p_m}{p_\infty}\right)^2 \right] \right\}^{-1}. \tag{6}$$

We shall estimate this quantity: for the initial equilibrium gas content $\bar{c}(0) = c_0$ and the pressure amplitude $(p_m/p_\infty)^2 = 0.2$, $2\sigma/p_\infty = 10^{-6}$ m, $D = 10^{-9}$ m²/sec, $c_0/\rho_g = 10^{-2}$ we have $T_3 = 100$ sec. Therefore, a universal bubble distribution can build up during several minutes in the sonicated medium, and become all the narrower near the mean critical radius whose location shifts towards the domain of large dimensions, the number of bubbles here decreases with time.

The solution represented describes the concluding stage in the evolution of a system of bubbles. As regards the initial stage, due to the limited amount of data about the distribution of cavitation nuclei and the mechanisms of their stabilization [6], we assume the most equally likely model, from our viewpoint, for microbubble stabilization in cracks of suspended solid particles. Coalescence holds here for bubbles with $R_0 \sim 5\cdot10^{-4}$ m if the (volume) concentration of the solid particles, the effective nuclei for this field magnitude, is $N > 2\cdot10^6$ m^{-3}. Indeed, by lowering the dissolved gas concentration by 10%, bubbles of the radius $R_0 = 5\cdot10^{-4}$ m "lay waste" to a domain of volume $V = 5\cdot10^{-7}$ m³ so that $VN > 1$.

In conclusion, we note that coalescence of gas bubbles was observed experimentally in supersaturated gelatin [7] in a study of the cavitation nuclei by optical and electron microscopy methods. The data represented in [7] graphically illustrate the process of dissolution of the smaller and growth of the larger of the two bubbles that turn out to be in close proximity.

LITERATURE CITED

1. G. Flynn, "Physics of acoustic cavitation in fluids," Physical Acoustics [Russian Translation], Vol. 1, Pt. B, Mir, Moscow (1967).
2. D. Y. Hsieh and M. Plesset, "Theory of rectified diffusion of mass into gas bubbles," JASA, 33, No. 2 (1961).
3. L. A. Crum and G. M. Hansen, "Generalized equations for rectified diffusion," JASA, 72, No. 5 (1982).
4. I. M. Lifshits and V. V. Slezov, "On the kinetics of diffusion dissociation of supersaturated solid solutions," Zh. Éksp. Teor. Fiz., 35, No. 2 (1958).
5. E. M. Lifshits and L. D. Pitaevskii, Physical Kinetics [in Russian], Nauka, Moscow (1979).
6. L. A. Crum, "Nucleation and stabilization of microbubbles in liquids," Appl. Sci. Res., 38, 101 (1982).
7. D. E. Yount, E. W. Gillary, and D. C. Hoffman, "A microscopic investigation of bubble formation nuclei," JASA, 76, No. 5 (1984).